

“LASER COOLING AND TRAPPING”

Phase Shifts of Atomic de Broglie Waves at an Evanescent Wave Mirror

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Abstract – A detailed theoretical investigation of the reflection of an atomic de Broglie wave at an evanescent wave mirror is presented. The classical and the semiclassical descriptions of the reflection process are reviewed, and a full wave-mechanical approach based on the analytical solution of the corresponding Schrödinger equation is presented. The phase shift at reflection is calculated exactly and interpreted in terms of instantaneous reflection of the atom at an effective mirror. Besides the semiclassical regime of reflection describable by the WKB method, a pure quantum regime of reflection is identified in the limit where the incident de Broglie wavelength is large compared to the evanescent wave decay length.

1. INTRODUCTION

Atomic mirrors are one of the key components in the field of atom optics [1]. In order to realize such a device, the use of evanescent waves appears very promising [2, 3]. In view of the future applications of evanescent wave mirrors, a detailed investigation of their optical properties seems appropriate. For some basic purposes (for example, the deflection of an atomic beam), the characterization of the geometric optical properties of the evanescent wave mirror is sufficient. This can be achieved by treating the incident atom as classical particles and by deriving their classical trajectories (this is analogous to the calculation of light rays in conventional optics). For more elaborate purposes (atom interferometers [4], atomic cavities [5, 6]), knowledge of the wave-mechanical properties of the evanescent wave mirror is also required. One then needs to describe the atom by a de Broglie wave in order to estimate the phase shift experienced by the atom during reflection at the mirror. A semiclassical derivation of this phase shift, based on the evaluation of the action integral along the classical atomic trajectories (WKB method), has been given by Opat *et al.* [7]. We present in this paper a more complete approach based on the analytical solution of the Schrödinger equation describing the interaction between the atom and the evanescent wave mirror in the regime of coherent atom optics (limit of negligible spontaneous emission). We interpret the atomic phase shift derived from the atomic wave function in terms of instantaneous reflection at an effective mirror, which generalizes the one introduced in [7]. We distinguish between a semiclassical

and a quantum regime of reflection. In the semiclassical regime, realized at high incident energy, the Schrödinger and the WKB approach coincide, and the evanescent wave mirror behaves as a dephasing dispersive mirror, analogous to a dielectric mirror in conventional optics. By contrast, in the quantum regime of reflection where the incident atomic de Broglie wavelength is large compared to the evanescent potential decay length, the evanescent wave mirror acts as a nondispersive infinitely steep barrier, analogous to a metallic mirror in conventional optics. Finally, the reflection process of an atomic wave packet incident on an evanescent wave mirror is discussed.

2. CLASSICAL AND SEMICLASSICAL DESCRIPTIONS OF THE REFLECTION PROCESS

Before turning to the full wave-mechanical treatment of atomic reflection at an evanescent wave mirror, we start by reviewing the classical dynamics as well as the WKB description of the reflection process. This will allow us to make a clear distinction between the semiclassical and the pure quantum features of atomic reflection.

2.1. Presentation of the Model

We consider the simple case of a two-level atom normally incident on the surface ($z = 0$ plane) of an evanescent wave mirror.¹ Because we are interested in the regime of coherent

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¹Because of the translational symmetry in the directions parallel to the mirror surface, the problem can be reduced to one dimension.

atom optics (limit of negligible spontaneous emission), we restrict ourselves to the limit of low saturation of the atomic transition, where the reactive part of the atom-evanescent wave coupling (light shift) is predominant over the dissipative part. We also assume that the detuning between the evanescent wave and the atomic frequency is properly chosen so that the atom can be considered to follow adiabatically the optical potential associated with the light-shifted ground-state level. In this regime, all the physical phenomena can be accounted for by means of the Hamiltonian [3, 7]:

$$H = \frac{p^2}{2M} + \frac{p_{\max}^2}{2M} \exp(-2\kappa z), \quad (1)$$

which contains the atomic kinetic energy (first term) and the reactive part of the atom-field coupling (second term). In equation (1), p and $z \geq 0$ are the momentum and position of the atomic center of mass, M is the atomic mass, $p_{\max} > 0$ is the maximum momentum that can be reflected by the optical potential barrier, and κ^{-1} is the characteristic decay length of the evanescent wave, of the order of the laser wavelength² (for a discussion of typical experimental parameters, see Appendix A). Note that when quantizing the atomic external degrees of freedom, one has to substitute the momentum and position operators P and Z for p and z in equation (1). The potential in equation (1) grows exponentially as $z \rightarrow -\infty$. We thus neglect any effects due to tunneling through the potential barrier to the physical mirror surface.

2.2. Classical Dynamics of the Reflection Process

Let us first consider the incident atom as a classical particle with asymptotic momentum $-p_\infty$ ($0 < p_\infty < p_{\max}$). With an appropriate choice of time origin, the classical trajectory of the atom can be written [7] (Fig. 1):

$$z(t) = z_0 + \kappa^{-1} \ln \cosh(t/\tau_{\text{refl}}), \quad (2)$$

where

$$z_0 = \kappa^{-1} \ln(p_{\max}/p_\infty) \quad (3)$$

is the position of the turning point of the trajectory (reached at $t = 0$), located about κ^{-1} in front of the mirror surface, and where

$$\tau_{\text{refl}} = M/\kappa p_\infty \quad (4)$$

is the time scale for the reflection process and corresponds to the time taken to cross the thickness κ^{-1} of the optical potential at the asymptotic velocity p_∞/M (see Appendix A, the table for typical experimental values).

sion. This simplification holds for both the classical and the quantum viewpoints.

²Note that, this estimate breaks down near the critical angle for the evanescent laser wave. In this case, the length scale κ^{-1} tends to infinity.

In the asymptotic region $z \gg \kappa^{-1}$ of vanishing optical potential, the atom is moving freely at constant velocity $\mp p_\infty/M$ along the asymptotes of the classical trajectory (see Fig. 1). These straight asymptotes intersect at the position

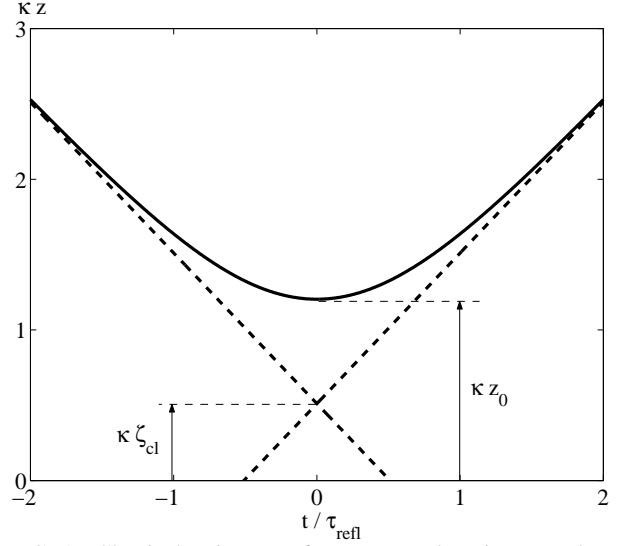


FIG. 1. Classical trajectory of an atom undergoing specular reflection at an evanescent wave mirror. The dimensionless atomic position κz is represented vs. t/τ_{refl} for the parameters $p_\infty = 3\hbar\kappa$ and $p_{\max} = 10\hbar\kappa$. The atom approaches the mirror surface at the minimum distance $z_0 \approx 1.2\kappa^{-1}$ and is reflected on a time scale of the order of τ_{refl} . In the asymptotic region ($\kappa z \gg 1$) of vanishing optical potential, the atom propagates freely at the velocity $\pm p_\infty/M$, and the classical trajectory corresponds to straight asymptotes (dashed lines), which intersect at the position $\zeta_{\text{cl}} \approx 0.5\kappa^{-1}$.

$$\zeta_{\text{cl}}(p_\infty) = z_0(p_\infty) - \kappa^{-1} \ln 2, \quad (5)$$

which is shifted more deeply into the potential relative to z_0 by a quantity independent of the incident momentum (see Fig. 1). As far as the asymptotic classical dynamics of the atom is concerned, the evanescent wave mirror thus behaves as a fictitious infinitely steep barrier located at $\zeta_{\text{cl}}(p_\infty)$, at which the atom would experience an instantaneous reflection. We name this barrier the classical effective mirror after [7].

2.3. The WKB Solution for the Evanescent Wave Mirror

Let us now consider the semiclassical description of the reflection process. In this case, the atom is described by means of a wave function derived using the WKB approximation. In

the classical allowed region $z > z_0$, this WKB wave function is given by [8]:

$$\begin{aligned}\psi_{\text{WKB}}(z) &= \sqrt{\frac{4M}{p(z)}} \sin\left(\frac{\pi}{4} + \frac{1}{\hbar} \int_{z_0}^z p(z') dz'\right) \\ &= \sqrt{\frac{4M}{p(z)}} \sin\left[\frac{\pi}{4} + \frac{p_\infty}{\hbar \kappa} (\text{artanh}(p(z)/p_\infty) - p(z)/p_\infty)\right],\end{aligned}\quad (6)$$

where $p(z)$ is the classical momentum calculated from energy conservation:

$$p(z)^2 + p_{\text{max}}^2 \exp(-2\kappa z) = p_\infty^2, \quad (7)$$

and where the phase $\pi/4$ results from the WKB connection formula, which matches the oscillating part of the wave function (6) to the decaying part in the classically forbidden region $z < z_0$. The normalization of the WKB wave function (6) has been chosen such that the incident and reflected waves both have unit flux independent of the asymptotic momentum.

In the asymptotic region, the atomic wave function is a superposition of two plane waves with wave vectors $k_\infty = \mp p_\infty/\hbar$, which correspond to the incident and reflected waves. The phase shift that we are interested in at reflection $\Delta\varphi_{\text{WKB}}$ is related to the relative phase between these two plane waves. We define $\Delta\varphi_{\text{WKB}}$ by writing the asymptotic WKB wave function in the form:

$$z \rightarrow +\infty : \psi_{\text{WKB}}(z) \cong \sqrt{\frac{4M}{p_\infty}} \sin\left(\frac{1}{\hbar} p_\infty z + \frac{1}{2} \Delta\varphi_{\text{WKB}}\right). \quad (8)$$

This definition of the phase shift takes as reference ($\Delta\varphi_{\text{WKB}} = 0$) a standing wave in front of an infinitely steep barrier located at the origin $z = 0$. It is important to note that this definition is somewhat arbitrary. In [7], for example, the reference is a standing wave in front of an infinitely steep barrier located at the position $z = \zeta_{\text{cl}}$ of the effective classical mirror [equation (5)]. Because ζ_{cl} depends on the incident atomic momentum, however, this phase reference is not absolute. With our definition of the phase shift, $\Delta\varphi_{\text{WKB}}$ is the phase correction in the situation where the evanescent optical potential is approximated by an infinitely steep barrier located at the mirror surface [6]. By writing the asymptotic expansion of the WKB wave function (6) in the form (8), one obtains [7]

$$\begin{aligned}\Delta\varphi_{\text{WKB}}(p_\infty) &= \frac{\pi}{2} - 2\frac{p_\infty}{\hbar \kappa} \left[1 + \ln\left(\frac{p_{\text{max}}}{2p_\infty}\right)\right] \\ &= \delta\varphi_{\text{WKB}} - 2p_\infty \zeta_{\text{WKB}}/\hbar\end{aligned}\quad (9)$$

³This decomposition separates the phase shift into a constant and an essentially linear term. Note that such an interpretation is not always unambiguous because one has to decompose $\Delta\varphi(p_\infty) = \delta\varphi - 2p_\infty \zeta/\hbar$, where $\delta\varphi \in [0, 2\pi]$ and ζ are weakly dependent on p_∞ .

$$\delta\varphi_{\text{WKB}} = \pi/2 \quad (10a)$$

$$\zeta_{\text{WKB}}(p_\infty) = \zeta_{\text{cl}}(p_\infty) + \kappa^{-1}. \quad (10b)$$

The order of magnitude of the WKB phase shift (9) is given by the ratio $p_\infty/\hbar\kappa$, which represents the phase shift associated with the free propagation of an atom of momentum p_∞ , through the spatial extent κ^{-1} of the evanescent optical potential.

The physical interpretation of (10) becomes transparent if we write the asymptotic WKB wave function (8) as³

$$\begin{aligned}z &\rightarrow +\infty : \\ \psi_{\text{WKB}}(z) &\cong \sqrt{\frac{4M}{p_\infty}} \sin\left(\frac{1}{2}\delta\varphi_{\text{WKB}} + \frac{1}{\hbar} p_\infty (z - \zeta_{\text{WKB}})\right),\end{aligned}\quad (11)$$

which corresponds to a plane standing wave whose phase is fixed to the value $\frac{1}{2}\delta\varphi_{\text{WKB}}$ at $z = \zeta_{\text{WKB}}$. As far as the asymptotic WKB wave function is concerned, the evanescent wave mirror is thus equivalent to an effective dephasing mirror located at the position ζ_{WKB} , where the atomic wave is instantaneously reflected and phase shifted by the amount $\delta\varphi_{\text{WKB}}$ (as light on a mirror). By analogy with the classical case (Section 2.2), we call this mirror the WKB effective mirror. The dephasing character of this mirror results from the WKB phase factor $\pi/4$ and thus has a nonclassical origin. It holds as long as the WKB approximation remains valid, i.e., as long as the incident de Broglie wavelength is small compared to the decay length κ^{-1} of the evanescent optical potential [8]. Furthermore, the evanescent wave mirror is dispersive because of the dependence of ζ_{WKB} on p_∞ . More precisely, the condition for the mirror to be dispersive is that $\partial^2 \Delta\varphi_{\text{WKB}}/\partial p_\infty^2 \neq 0$: a linear dependence of $\Delta\varphi_{\text{WKB}}$ on p_∞ can always be removed by an appropriate choice of absolute phase reference. Finally, it is interesting to note that the position ζ_{WKB} [equation (10b)] of the WKB effective mirror differs from the classical effective mirror position ζ_{cl} [equation (5)] by the quantity κ^{-1} independent of the incident atomic momentum. As a result, the classical and the WKB description of the reflection process yield comparable physical pictures.

3. SCHRÖDINGER WAVE FUNCTION APPROACH

We now turn to the full wave-mechanical description of atomic reflection at the evanescent wave mirror. This description is based on the analytical solution of the corresponding

Schrödinger equation, which allows an exact calculation of the phase shift at reflection.

3.1. Solution of the Stationary Schrödinger Equation

The full quantum description of atomic reflection consists in solving exactly the stationary Schrödinger equation for the atomic wave function $\psi(z)$:

$$\left(-\hbar^2 \frac{d^2}{dz^2} + p_{\max}^2 \exp(-2\kappa z)\right) \psi(z) = p_{\infty}^2 \psi(z). \quad (12)$$

We use the change of variable

$$z \rightarrow u = \frac{p_{\max}}{\hbar\kappa} \exp(-\kappa z) \quad (13)$$

which takes advantage of the invariance of the Hamiltonian (1) under the transformation

$$\forall a, \quad \begin{cases} z \rightarrow z + a \\ p_{\max} \rightarrow e^{2\kappa a} p_{\max} \end{cases}. \quad (14)$$

Equation (12) transforms into a Bessel-type equation:

$$\left(u^2 \frac{d^2}{du^2} + u \frac{d}{du} - (u^2 - \alpha^2)\right) \psi(u) = 0, \quad (15)$$

which only depends on one dimensionless parameter:

$$\alpha = p_{\infty}/\hbar\kappa. \quad (16)$$

The solutions of (15) are linear combinations of the Bessel functions $I_{\pm i\alpha}(u)$. Two boundary conditions impose a unique solution:

(i) The wave function must vanish in the limit $z \rightarrow -\infty$ (the probability of the atoms being in the region $z \leq z_0$ inside the potential being small compared to the probability of being in the classically allowed region $z \geq z_0$).

(ii) In the asymptotic region $z \rightarrow +\infty$, the wave function is normalized in the same way as the WKB solution [equation (6)].

As shown in Appendix B, these conditions lead to the solution⁴

$$\psi_{\text{Schr}}(z) = \sqrt{\frac{4M}{p_{\infty} \sinh(\pi\alpha)}} \frac{1}{2i} (I_{-i\alpha}(u(z)) - I_{i\alpha}(u(z))). \quad (17)$$

The Schrödinger wave function (17) and the corresponding WKB wave function (6) are represented in Fig. 2 as a function

of the dimensionless parameter κz , in the case $p_{\infty} = 3\hbar\kappa$ and $p_{\max} = 10\hbar\kappa$. One sees that the wave functions are in good agreement in both the asymptotic region ($\kappa z \gg 1$) and far inside the optical potential ($\kappa z \ll 1$), but that they significantly differ around the classical turning point $\kappa z_0 \cong 1.2$ (where the WKB wave function actually diverges).

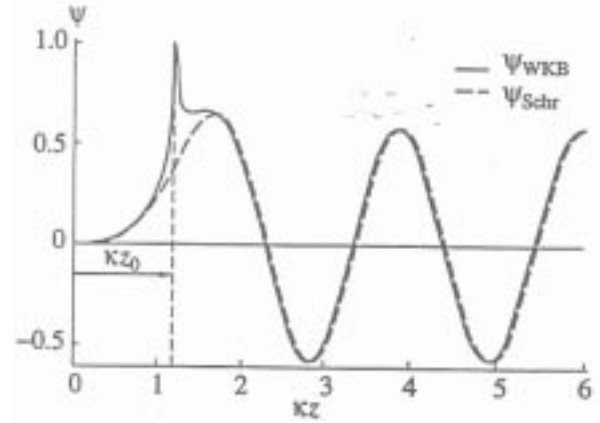


FIG. 2. Comparison between the WKB (ψ_{WKB}) and the Schrödinger (ψ_{Schr}) wave functions for the same parameters as in Fig.1. These wave functions coincide both in the asymptotic region ($\kappa z \gg 1$) and far inside the optical potential ($\kappa z \ll 1$) but significantly differ around the classical turning point z_0 , where ψ_{WKB} diverges.

3.2. Phase Shift of the Schrödinger Wave Function

The exact solution (17) of the Schrödinger equation allows us to derive exactly the phase shift experienced by the atomic wave function at reflection on the evanescent wave mirror.

Following the definition (8) of the WKB phase shift at reflection, we define the Schrödinger phase shift $\Delta\varphi_{\text{Schr}}$ by writing the asymptotic Schrödinger wave function (17) in the form:

$$z \rightarrow +\infty : \psi_{\text{Schr}}(z) \cong \sqrt{\frac{4M}{p_{\infty}}} \sin\left(\frac{1}{\hbar} p_{\infty} z + \frac{1}{2} \Delta\varphi_{\text{Schr}}\right). \quad (18)$$

By using (17) and the asymptotic expansion ($u \rightarrow 0$) of the Bessel functions $I_{\pm i\alpha}(u)$, one obtains (see Appendix B)

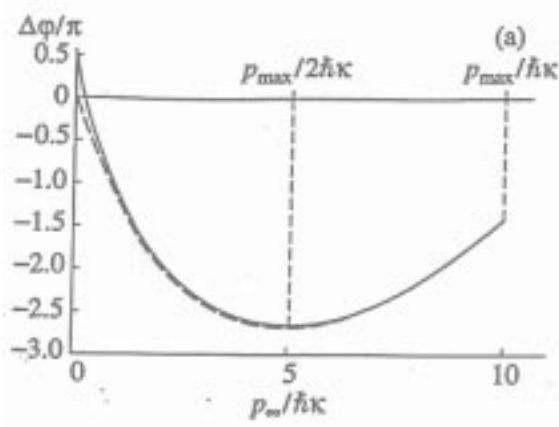
$$\Delta\varphi_{\text{Schr}}(p_{\infty}) = -2\alpha \ln\left(\frac{p_{\max}}{2\hbar\kappa}\right) + 2 \arg \Gamma(1 + i\alpha), \quad (19)$$

⁴We have neglected any loss resulting from atomic tunneling to the mirror surface. However, the tunneling probability can be estimated by the flux of the wave function at $z = 0$. *Note added after publication:* To our knowledge, the exact solution (17) for the exponential barrier has been first derived by J. M. Jackson and N. F. Mott in 1932, *Proc. Roy. Soc. (London) Ser. A* **137**, 703.

where Γ is the Euler gamma (factorial) function [9], and where $\arg \Gamma(1 + i\alpha)$ is the argument of the complex number $\Gamma(1 + i\alpha)$ defined as a continuous function of α .

The exact ($\Delta\varphi_{\text{Schr}}$) and the semiclassical ($\Delta\varphi_{\text{WKB}}$) phase shifts at reflection are represented in Fig. 3 as a function of the dimensionless parameter $\alpha = p_{\infty}/\hbar\kappa$. One can clearly distinguish between two limiting cases.

In the limit $\alpha \gg 1$ (high incident momentum), where the incident atomic de Broglie wavelength is small compared to the decay length κ^{-1} of the optical potential, the WKB and the Schrödinger approaches yield comparable phase shifts [8].



Equations (21) and (22) show that, in the quantum regime of reflection, the evanescent wave mirror behaves as a nondephasing effective mirror located at the position $z \approx \zeta_{cl}(\hbar\kappa)$

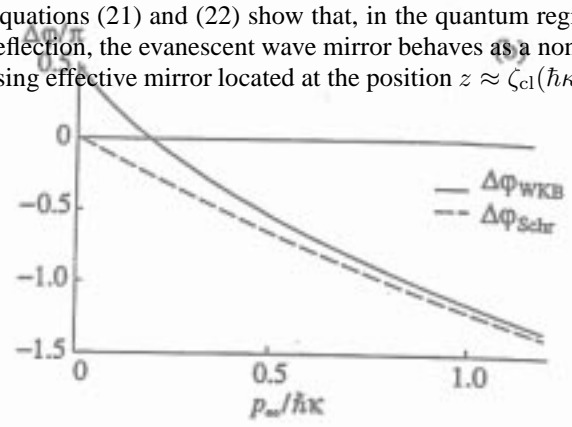


FIG. 3. Dependence of the WKB ($\Delta\varphi_{\text{WKB}}$) and of the Schrödinger ($\Delta\varphi_{\text{Schr}}$) phase shifts vs. the dimensionless incident atomic momentum $\alpha = p_{\infty}/\hbar\kappa$ for $p_{\text{max}} = 10\hbar\kappa$. (a) $\Delta\varphi_{\text{WKB}}$ and $\Delta\varphi_{\text{Schr}}$ coincide in the limit of high incident atomic momentum ($\alpha \gg 1$), which thus corresponds to a semiclassical regime of reflection. (b) In the limit of low incident atomic momentum ($\alpha \ll 1$), the semiclassical description of the evanescent wave mirror does not yield the correct phase shift at reflection. This corresponds to a pure quantum regime of reflection.

This corresponds to the semiclassical regime of atomic reflection considered in Section 2.3. Using equation (19), it is possible to derive the first correction to the semiclassical phase shift. One thus finds [9]

$$\alpha \gg 1 : \Delta\varphi_{\text{Schr}}(p_{\infty}) = \Delta\varphi_{\text{WKB}} - \frac{1}{6\alpha} + O(\alpha^{-3}). \quad (20)$$

In the limit $\alpha \ll 1$ (low incident momentum), where the incident atomic de Broglie wavelength is larger than the decay length of the evanescent optical potential, the WKB and the Schrödinger approaches yield different phase shifts. In particular, in the limit $\alpha \rightarrow 0^+$, $\Delta\varphi_{\text{WKB}}$ tends toward $\pi/2$, whereas $\Delta\varphi_{\text{Schr}}$ tends toward 0 (see Fig. 3b). The limit of low incident momentum thus corresponds to a pure quantum regime of reflection, which can not be appropriately described in semiclassical terms. In order to obtain in this regime a representation of the evanescent wave mirror in terms of an effective mirror, we write equation (19) in a form analogous to (9). One thus finds [9]

$$\begin{aligned} \alpha \ll 1 : \\ \Delta\varphi_{\text{Schr}} &\cong -2\alpha \left(\gamma + \ln \left(\frac{p_{\text{max}}}{2\hbar\kappa} \right) \right) \\ &= \delta\varphi_{\text{Schr}} - 2p_{\infty}\zeta_{\text{Schr}}/\hbar \end{aligned} \quad (21)$$

with the Euler constant $\gamma \cong 0.577$ and

$$\delta\varphi_{\text{Schr}} \cong 0 \quad (22a)$$

$$\zeta_{\text{Schr}}(p_{\infty} \ll \hbar\kappa) \cong \zeta_{cl}(\hbar\kappa) + \gamma\kappa^{-1}. \quad (22b)$$

[equation (22b)], where the atomic wave is instantaneously reflected (as light on a metallic mirror). The nondephasing character of the Schrödinger effective mirror [equation (22a)] results from the fact that, on the spatial scale of the incident de Broglie wavelength, the evanescent optical potential appears to be an infinitely steep barrier. In the quantum regime of reflection, it is therefore legitimate to approximate the evanescent wave mirror by a hard barrier located at the position of the classical effective mirror for an asymptotic momentum $p_{\infty} \approx \hbar\kappa$. It is also interesting to note that, contrary to the semiclassical case, the position ζ_{Schr} of the Schrödinger effective mirror is essentially independent of p_{∞} . As a result, the evanescent wave mirror is no longer dispersive in the quantum regime.⁵

4. REFLECTION OF AN ATOMIC WAVE PACKET

In the experiments using effusive beams as a source of atoms, it is possible to describe the incident particles in terms of statistical mixtures of de Broglie waves having a well-defined momentum (plane waves). In that case, the reflection process at the evanescent wave mirror can be directly characterized using the results of the preceding sections. However, in some other situations (for example, when the incident

⁵It is perhaps surprising that the evanescent wave mirror is not dispersive while the Schrödinger phase shift (21) depends linearly on the incident momentum. In fact, this dependence is related to the choice of reference for the phase shift. Thus, by taking as phase reference a standing wave in front of an infinitely steep barrier located in $z \cong \zeta_{cl}(\hbar\kappa) + \gamma\kappa^{-1}$, the phase shift at reflection in the quantum regime would be independent of the incident momentum.

atoms originate from an optical molasses where atom localization takes place [10]), it is more appropriate to describe the particles in terms of wave packets (superpositions of plane waves). In such a case, each partial plane wave of incident momentum p_∞ experiences a different phase shift at reflection $\Delta\varphi_{\text{Schr}}(p_\infty)$ (as given in Section 3.2), which shows up in a spatial shift of the center of the wave packet.

Let us consider an atomic wave packet incident on an evanescent wave mirror. In the asymptotic region, one may write the incident part of the atomic wave function $\psi_{\text{inc}}(z, t)$ as

$$\psi_{\text{inc}}(z, t) = \int dp_\infty \tilde{\psi}_{\text{inc}}(p_\infty) \exp\left(-i\frac{p_\infty^2 t}{2M\hbar} - i\frac{p_\infty z}{\hbar}\right), \quad (23)$$

where $\tilde{\psi}_{\text{inc}}(p_\infty)$ denotes the Fourier transform of ψ_{inc} . During the reflection process, each partial plane wave experiences a different phase shift $\Delta\varphi_{\text{Schr}}(p_\infty)$ [equation (19)], so that in the asymptotic region, the reflection part of the atomic wave function $\psi_{\text{ref}}(z, t)$ reads

$$\psi_{\text{ref}}(z, t) = - \int dp_\infty \tilde{\psi}_{\text{inc}}(p_\infty) \times \exp\left(-i\frac{p_\infty^2 t}{2M\hbar} + i\frac{p_\infty z}{\hbar} + i\Delta\varphi_{\text{Schr}}(p_\infty)\right), \quad (24)$$

where the minus sign in front of the integral results from our peculiar choice of phase origin.

By assuming that $\tilde{\psi}_{\text{inc}}(p_\infty)$ is peaked around the average momentum \bar{p}_∞ , it is possible to characterize the position $z_{\text{wp}}(t)$ of the center of the wave packet (23) or (24) via the method of stationary phase. One readily finds

$$z_{\text{wp}}^{\text{inc}}(t) = -\frac{\bar{p}_\infty}{M}t, \quad (25a)$$

$$z_{\text{wp}}^{\text{ref}}(t) = -\frac{\bar{p}_\infty}{M}t - \hbar \left(\frac{\partial \Delta\varphi_{\text{Schr}}(p_\infty)}{\partial p_\infty} \right)_{\bar{p}_\infty}, \quad (25b)$$

where $z_{\text{wp}}^{\text{inc}}(z_{\text{wp}}^{\text{ref}})$ denotes the position of the center of the incident (reflected) wave packet. Equation (25) shows that, as far as the asymptotic wave packets are concerned, the evanescent wave mirror behaves as an infinitely steep effective mirror located at the position ζ_{wp} given by

$$\zeta_{\text{wp}} = -\frac{\hbar}{2} \left(\frac{\partial \Delta\varphi_{\text{Schr}}}{\partial p_\infty} \right)_{\bar{p}_\infty}. \quad (26)$$

Substituting for $\Delta\varphi_{\text{Schr}}$ in (26) using (19) gives

$$\zeta_{\text{wp}} = \kappa^{-1} (\ln(p_{\text{max}}/2\hbar\kappa) - \text{Re}\Psi(1 + i\bar{\alpha})), \quad (27)$$

where $\bar{\alpha} = \bar{p}_\infty/\hbar\kappa$ is the dimensionless parameter given by equation (16), and where Ψ is the digamma function defined as

$$\Psi(x) = \partial \ln \Gamma(x) / \partial x. \quad (28)$$

As in the preceding section, we distinguish between two limiting regimes of reflection of the atomic wave packet.

In the limit $\bar{\alpha} \gg 1$ (semiclassical regime of reflection, see Section 3.2), where equation (27) reduces to

$$\bar{\alpha} \gg 1 : \zeta_{\text{wp}} = \zeta_{\text{cl}}(\bar{p}_\infty) + O(1/\bar{\alpha}^2), \quad (29)$$

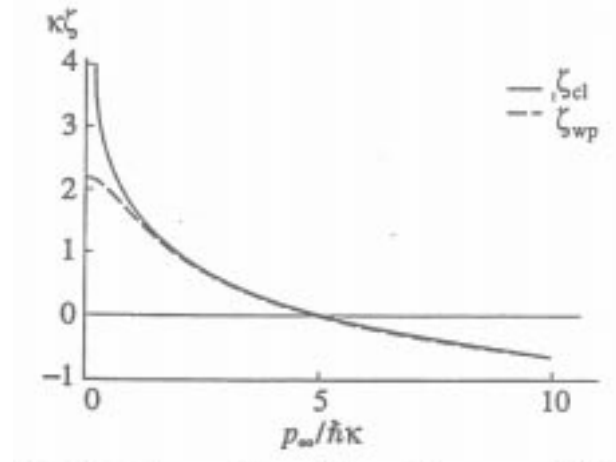
the asymptotic wave packet appears to be instantaneously reflected at an effective mirror located in $z = \zeta_{\text{cl}}$ [equation (5)]. The reflection process is thus analogous to that of a classical particle of incident momentum \bar{p}_∞ .⁶ Note that the position of the effective mirror for the atomic wave packet corresponds to ζ_{cl} , and not to ζ_{WKB} [equation (10b)].

In the limit $\bar{\alpha} \ll 1$ (quantum regime of reflection), where equation (27) reads

$$\bar{\alpha} \ll 1 : \zeta_{\text{wp}} \cong \zeta_{\text{Schr}}(\bar{\alpha} \ll 1) \cong \zeta_{\text{cl}}(\hbar\kappa) + \gamma\kappa^{-1}, \quad (30)$$

the evanescent wave mirror behaves as an infinitely steep potential barrier located at the position $z \approx \zeta_{\text{cl}}(\hbar\kappa) + \gamma\kappa^{-1}$, which instantaneously reflects the atomic wave packet. The fact that this position coincides with that of the effective mirror for an atomic plane wave of incident momentum \bar{p}_∞ [equation (22b)] is not surprising because in the quantum regime, the position of the effective mirror ζ_{Schr} is independent of the incident atomic momentum (see Section 3.2). As a result, all partial plane waves of the wave packet are reflected at the same barrier, and the wave packet behaves in the same way.

The position ζ_{wp} of the effective mirror describing the reflection process of the wave packet at the evanescent wave mirror is represented in Fig. 4 together with ζ_{cl} as a function of the dimensionless parameter $\bar{\alpha} = \bar{p}_\infty/\hbar\kappa$. One clearly distinguishes between the semiclassical and the quantum regime of reflection of the wave packet.



⁶Within the framework of the WKB approach, it can be shown that this result holds for any mirror potential vanishing at large z .

FIG. 4. Dependence of the positions of the wave packet (ζ_{wp}) and the classical (ζ_{cl}) effective mirrors vs. the dimensionless average incident momentum of the wave packet $\bar{\alpha} = \bar{p}_{\infty}/\hbar\kappa$ for $p_{\text{max}} = 10\hbar\kappa$. ζ_{cl} and ζ_{wp} coincide in the semiclassical regime of reflection ($\bar{\alpha} \gg 1$). By contrast, in the quantum regime of reflection ($\bar{\alpha} \ll 1$), ζ_{wp} tends toward a constant, whereas ζ_{cl} tends to infinity as $\bar{\alpha} \rightarrow 0^+$.

5. CONCLUSION

We have presented a detailed theoretical investigation of the reflection process of an atomic de Broglie wave at an evanescent wave mirror in the regime of coherent atom optics. Our calculation of the atomic phase shift at reflection using the exact solution of the corresponding Schrödinger equation has allowed us to identify two limiting regimes of reflection. The semiclassical regime corresponds to incident de Broglie wavelengths much smaller than the decay length of the evanescent optical potential and can be satisfactorily accounted for by the WKB method. The evanescent wave mirror then behaves as a dispersive dephasing mirror. In the quantum regime of reflection, where the incident atomic de Broglie wavelength is larger than the decay length of the evanescent potential, the evanescent wave mirror behaves as a nondispersive hard potential barrier located in front of the actual evanescent wave mirror surface.

In experiments using either a supersonic beam or laser-cooled atoms accelerated by the earth gravity field, the minimum achievable atomic incident momentum is typically of the order of $10\hbar\kappa$. Under such conditions, the atomic reflection process can always be accounted for in semiclassical terms. However, the experimental observation of atomic reflection in the quantum regime seems feasible, using, for example, an evanescent wave mirror located at the summit of an atomic fountain where the atomic momentum approaches zero. Another, more challenging possibility would be to investigate the lowest bouncing modes of a gravitational cavity [6], which correspond to a de Broglie wavelength of the order of the decay length of the evanescent optical potential, and thus realize the quantum regime of reflection for the bouncing atoms.

The exact derivation of the atomic wave function presented in this paper [equation(17)] can serve as a starting point for the investigation of many other effects. These include tunneling through the optical potential barrier, the influence of atomic internal states on the reflection process, and, especially interesting, dipole-surface effects (such as the Van der Waals interaction), which may modify the potential we have assumed here, and hence also the phase shift at reflection.

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APPENDIX A: EXPERIMENTAL INVESTIGATION OF THE REFLECTION PROCESS

We would like to comment in this appendix on the typical experimental parameters required to observe atomic reflection on an evanescent wave mirror. The regime of coherent atom optics is realized provided that the probability of spontaneous emission during reflection is negligible. It corresponds to the limit of small saturation of the atomic transition:

$$s = \frac{\Omega^2/2}{\Delta^2 + \Gamma^2/4} \ll 1, \quad (\text{A1})$$

where $\Omega = -dE_0/\hbar$ is the resonant Rabi frequency that characterizes the coupling between the atomic dipole d and the evanescent field of maximum amplitude E_0 (at the evanescent wave mirror surface), Γ is the natural width of the atomic excited state, and $\Delta = \omega - \omega_A$ is the detuning between the frequency (ω) of the evanescent wave and the atomic (ω_A) frequency. It also requires that the incident atom follows adiabatically the optical potential associated with the light-shifted ground-state level. This is achieved in the limit of large detuning from resonance (we assume $\Delta > 0$, which allows an atom entering the optical potential in the ground-state to be reflected at the evanescent wave mirror):

$$\Delta \gg \Gamma \quad (\text{A2})$$

In the regime where (A1) and (A2) are fulfilled, it is possible to describe the atom reflection process by means of the Hamiltonian (1). Note, however, that for a given laser intensity, the maximum reflectible atomic momentum p_{max} decreases as the frequency detuning increases, as shown by the relation

$$\frac{p_{\text{max}}^2}{2M} = \frac{1}{2}\hbar\Delta s \quad (\text{A3})$$

In fact, by designing the evanescent wave mirror with the multilayer coating technique [12], it is possible to fulfill conditions (A1) and (A2) while simultaneously being able to reflect atoms having high incident momentum ($p_{\text{max}} \gg \hbar\kappa$). Typical experimental parameters are indicated in the table in the case of the D2 line of ^{85}Rb .

Typical experimental parameters for atomic reflection at an evanescent wave mirror using the D2 line of ^{85}Rb atoms

Physical parameters	Notation	Typical value
Laser wavelength		780 nm
Natural linewidth	Γ	6 MHz
Frequency detuning from resonance	Δ	$5 \times 10^4 \Gamma$
Maximum intensity of the evanescent wave	E_0^2	10^4 W/cm^2
Saturation parameter	s	6×10^{-4}
Maximal reflected momentum	p_{\max}	$150 \hbar \kappa$
Probability of spontaneous emission per reflection for $p_{\infty} = p_{\max}$		2.5×10^{-3}
Probability of nonadiabatic departure from the light-shifted ground-state level for $p_{\infty} = p_{\max}$		$\leq 8 \times 10^{-15}$
Decay length of the evanescent optical potential	$1/\kappa^{-1}$	$\approx 100 \text{ nm}$
Reflection time for $p_{\infty} = p_{\max}$	τ_{refl}	$\approx 4 \Gamma^{-1}$

APPENDIX B: SOLUTION OF THE STATIONARY SCHRÖDINGER EQUATION

The general solution of the Bessel-type Schrödinger equation (15) is a linear combination of the Bessel functions $I_{\pm i\alpha}(u)$. These functions are defined by [11]

$$I_{\pm i\alpha} = \sum_{n=0}^{\infty} \frac{1}{n! \Gamma(n+1 \pm i\alpha)} \left(\frac{u}{2}\right)^{2n \pm i\alpha}, \quad (\text{B1})$$

where Γ denotes the Euler gamma function. They both diverge as $z \rightarrow -\infty \Leftrightarrow u \rightarrow +\infty$ according to [11]

$$u \rightarrow +\infty : I_{\pm i\alpha}(u) \cong \frac{1}{\sqrt{2\pi u}} e^u (1 + O(1/u)). \quad (\text{B2})$$

As a result, the only linear combination of $I_{\pm i\alpha}(u)$ satisfying the boundary condition (i) of Section 3.1 corresponds to the difference of $I_{i\alpha}(u)$ and $I_{-i\alpha}(u)$. This difference is proportional to the Bessel- K function:

$$K_{i\alpha} = \frac{\pi}{\sinh(\pi\alpha)} \frac{1}{2i} (I_{-i\alpha}(u) - I_{i\alpha}(u)), \quad (\text{B3})$$

whose asymptotic expansion is [11]:

$$u \rightarrow +\infty : K_{i\alpha}(u) \cong \sqrt{\frac{\pi}{2u}} e^{-u} (1 + O(1/u)). \quad (\text{B4})$$

Equation (B4) shows that the atomic wave function decays very rapidly (as $\exp[-(p_{\max}/\hbar\kappa)e^{-\kappa z}]$) inside the potential barrier.

We finally consider the boundary condition (ii) of the Section 3.1. In the asymptotic region $z \rightarrow +\infty \Leftrightarrow u \rightarrow 0^+$, the expansion of $I_{\pm i\alpha}(u)$ is given by the first term of the series expansion (B1):

$$z \rightarrow +\infty : I_{\pm i\alpha}(u(z)) \cong \frac{1}{|\Gamma(1+i\alpha)|} \exp(\mp i p_{\infty} z / \hbar) \pm i\alpha \ln\left(\frac{p_{\max}}{2\hbar\kappa}\right) \mp i \arg \Gamma(1+i\alpha) \quad (\text{B5})$$

with [9]:

$$|\Gamma(1+i\alpha)| = \sqrt{\frac{\pi\alpha}{\sinh(\pi\alpha)}}. \quad (\text{B6})$$

Combining (B3), (B5), and (B6), one readily finds that the only solution of equation (15) satisfying the boundary conditions (i) and (ii) of Section 3.1 is:

$$\psi_{\text{Schr}}(z) = \sqrt{\frac{4M}{p_{\infty}} \frac{\pi\alpha}{\sinh(\pi\alpha)}} \frac{1}{2i} (I_{-i\alpha}(u(z)) - I_{i\alpha}(u(z))), \quad (\text{B7})$$

or equivalently

$$\psi_{\text{Schr}}(z) = \sqrt{\frac{4M}{\pi\hbar\kappa}} \sinh(\pi\alpha) K_{i\alpha}(u(z)). \quad (\text{B8})$$

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